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Conformational Analysis via Chirotope Generation

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Conformational analysis

A chemical structure ...



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Conformational analysis

A chemical structure ...



cyclohexane

A chemical structure ...



... may appear in different *conformations*.



A chemical structure ...



... may appear in different conformations.





chair form

twisted form



A chemical structure ...



... may appear in different conformations.





Under *conformer generation*, we understand the generation of a reasonably distributed sample of the conformation space.

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The orientation function

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The orientation function



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The orientation function



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The orientation function

• The "right-hand rule":



• In molecules:



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The orientation function

• The "right-hand rule":





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The orientation function

• The "right-hand rule":





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The orientation function

• The "right-hand rule":





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The orientation function

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~2²³⁴ +

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The orientation function

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The orientation function





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The orientation function





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The orientation function





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The orientation function

• The "right-hand rule":





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The orientation function

• The "right-hand rule":





b

The orientation function

• The "right-hand rule":



The orientation function χ (in combination with the molecular graph) describes a molecule on an intermediate level between constitution and conformation.

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Conformer generation

Conformer generation

Our Strategy for conformer generation:



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Conformer generation

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• Generate mappings, which potentially are orientation functions.

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Conformer generation

Our Strategy for conformer generation:

- Generate mappings, which potentially are orientation functions.
- Try to find a conformer for each of these mappings.

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As potential orientation functions, we consider chirotopes.



Chirotope

A chirotope (of rank 4) over *n* points (*the atoms*) is a mapping

$$\chi: n^4 \rightarrow \{0, \pm 1\}$$

with:



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$$\chi(\mathbf{a}_{\pi^{-1}(\mathbf{0})},\ldots,\mathbf{a}_{\pi^{-1}(\mathbf{3})}) = \operatorname{sgn}(\pi) \cdot \chi(\mathbf{a}_0,\ldots,\mathbf{a}_3).$$

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• χ fulfills the binary Grassmann-Plücker relations:
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• χ fulfills the binary Grassmann-Plücker relations:

$$\chi(\vec{a}) \cdot \chi(\vec{b}) = 1 \Longrightarrow$$

$$\exists i \in n : \chi(b_i, a_1, \dots, a_3) \cdot \chi(b_0, \dots, a_0, \dots, b_3) = 1.$$
(GP)

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Canonical forms of chirotopes



• Canonical forms are important for chirotopes (as they are for the molecular graphs).

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- Canonical forms are important for chirotopes (as they are for the molecular graphs).
- We are able to calculate the canonical form.

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Radon partitions

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Radon partitions



Radon partitions



Radon partitions

• A chemical unfeasable conformation:



Question:

Is it possible to recognize the infeasibility from the orientation function only?

Radon partitions (2)

• A *radon partition* is a pair (A,B) of subsets of all atoms, such that their convex hulls intersect:

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• The chirotope determines all radon partitions.



Radon partitions (2)

• A *radon partition* is a pair (A,B) of subsets of all atoms, such that their convex hulls intersect:



- The chirotope determines all radon partitions.
- This way, we can recognize (some) chemical unfeasable configurations, e.g.



Radon partitions



Radon partitions



Radon partitions



Radon partitions



Radon partitions



Radon partitions



Radon partitions

• A chemical unfeasable conformation:



• We did not need coordinates nor angles for this test.

Radon partitions



- We did not need coordinates nor angles for this test.
- \Rightarrow Efficient test during conformer generation.

















































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Partially defined chirotopes





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Partially defined chirotopes





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Partially defined chirotopes



• Partially defined chirotopes give the possibility to classify the conformations in a graduated application-specific manner.



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• The molecular graph has 12 automorphisms.



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- The molecular graph has 12 automorphisms.
- We assume, that no 4 atoms are in a plane.



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- Excluding unfeasable radon partitions: 162





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- Excluding unfeasable radon partitions: 162
- Restricting to quadrupels corresponding to a gauche/anti-situation:



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- Conformation as local minima of an energy function were found for:





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- Excluding unfeasable radon partitions: 162
- Restricting to quadrupels corresponding to a gauche/anti-situation: 13
- Conformation as local minima of an energy function were found for: 4



23A	224226	1250	14503450
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- The molecular graph has 12 automorphisms.
- We assume, that no 4 atoms are in a plane.
- We get <u>386</u> chirotopes.
- Excluding unfeasable radon partitions: 162
- Restricting to quadrupels corresponding to a gauche/anti-situation: 13
- Conformation as local minima of an energy function were found for: 4
 - boat form (a "sattle point")



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- The molecular graph has 12 automorphisms.
- We assume, that no 4 atoms are in a plane.
- We get <u>386</u> chirotopes.
- Excluding unfeasable radon partitions: 162
- Restricting to quadrupels corresponding to a gauche/anti-situation: 13
- Conformation as local minima of an energy function were found for: 4
 - boat form (a "sattle point")
 - twist form



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- The molecular graph has 12 automorphisms.
- We assume, that no 4 atoms are in a plane.
- We get 386 chirotopes.
- Excluding unfeasable radon partitions: 162
- Restricting to quadrupels corresponding to a gauche/anti-situation: 13
- Conformation as local minima of an energy function were found for: 4
 - boat form (a "sattle point")
 - twist form
 - chair form



23A	224236	1250	1450 3450
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- Excluding unfeasable radon partitions: 162
- Restricting to quadrupels corresponding to a gauche/anti-situation: 13
- Conformation as local minima of an energy function were found for: 4
 - boat form (a "sattle point")
 - twist form
 - chair form
 - twist form (enantiomere)



23A	224220	1250	1450 3450
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Thank You!