# Constructing Two-weight Codes with Prescribed Groups of Automorphisms

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## Introduction

We are interested in the construction of linear $[n, k; q]$ two-weight codes. A linear code is a $k-$dimensional subspace $C$ of the $n-$dimensional vector space $GF(q)^n$ over the finite field $GF(q)$ with $q$ elements. The $q^k$ codewords of length $n$ are the elements of the subspace, they are written as row vectors. The weight of a codeword $c$ is the number of nonzero components of the vector $c \in GF(q)^n$. In the case of a two-weight code $C$ the nonzero elements of $C$ have only two different weights $w_1$ and $w_2$ with $w_1 < w_2$.

Two-weight codes are an interesting object, as there are connections to objects in different areas of mathematics like strongly regular graphs, partial geometries and projective point sets. But two-weight codes are also interesting in the area of coding theory itself (e.g. uniformly packed codes) and have been studied intensively [11]. Delsarte [12] was the first to study the connections between two-weight codes, strongly regular graphs and projective point-sets. A survey of this relationship was given later by Calderbank and Kantor [10].

**Projective Point Sets**

To formulate a two-weight code as a solution of a Diophantine system of equations we use in a first step a well known equivalence between linear two-weight $[n, k; q]$ codes and point sets in the projective space $PG(k-1, q)$. A linear $[n, k; q]$ code $C$ is described by a generator matrix, i.e. a $k \times n$ matrix over $GF(q)$ whose row-space is the subspace $C$. If we assume that the columns of a generator matrix are pairwise linearly independent, we can take the columns of the generator matrix as a set (no multiple points as columns are linearly independent) of points in $PG(k-1, q)$. Because of this correspondence such a linear code is called *projective*. Using this correspondence it is well-known [10] that the weights of the code become the intersection numbers between the projective point set and the hyperplanes of $PG(k-1, q)$. In the case of a two-weight code the corresponding point set $\Omega$ has the property that every hyperplane meets $\Omega$ in $n - w_1$ or $n - w_2$ points. Such a point set is called a $(n, k, n - w_1, n - w_2)$ point set. To construct projective two-weight codes we construct the corresponding point sets.

The next step is to formulate the construction of the point set as a solution of a Diophantine system of linear equations. For this let $M$ be the point - hyperplane incidence matrix of $PG(k-1, q)$. Incidence is given by the subset relation between subspaces of $GF(q)^k$. $M$ is a $p \times p$ square matrix, where $p$ is the number of points. The columns are labeled by the points and rows are labeled by the hyperplanes. This incidence matrix $M$ can be used to formulate the construction of the point set as a solution of a system of equations.

**Theorem 1** *There is an* $(n, k, n - w_1, n - w_2)$ *point set in* $PG(k - 1, q)$ *if and only if there is a* $(0/1)-$*solution* $x = (x_1, \ldots, x_p)$ *of the*

*Diophantine system of* $p+1$ *linear equations:*

$$(1) \quad Mx^T = \begin{pmatrix} n-w_1 & \textit{or} & n-w_2 \\ & \vdots & \\ n-w_1 & \textit{or} & n-w_2 \end{pmatrix}$$

$$(2) \ \textstyle\sum_{i=1,\ldots,p} x_i = \qquad n.$$

An entry $x_i$ equal to one in the solution says that the corresponding point labeling the $i-$th column of $M$ is part of the $(n, k, n-w_1, n-w_2)$ point set. The first $p$ equations given in a closed matrix notation ensure that there are only two intersection numbers. The last equation ensures that point set has order $n$.

To solve this system using the computer we transfer it in the following form, where $J$ is a $p \times p$ diagonal matrix with the entry $(w_2 - w_1)$ on the diagonal:

**Corollary 2** *There is an* $(n, k, n-w_1, n-w_2)$ *point set in* $PG(k-1, q)$ *if and only if there is a* $(0/1)-$*solution* $(x, y) = (x_1, \ldots, x_p, y_1, \ldots, y_p)$ *of the Diophantine system of* $p+1$ *linear equations:*

$$(1) \ (M, J)(x, y)^T = \begin{pmatrix} n-w_1 \\ \vdots \\ n-w_1 \end{pmatrix}$$

$$(2) \quad \textstyle\sum_{i=1,\ldots,p} x_i \quad = \qquad n.$$

Here $(M, J)$ denotes the $p \times 2p$ block matrix built from matrices $M$ and $J$. An entry $y_i$ equal to one says that the $i-$th point (which is in the point set given by the solution $x$) is met by $n-w_2$ hyperplanes, an entry $y_i$ equal to zero says that this point is met by $n-w_1$ hyperplanes.

The limiting factor for computation of a solution is the size (=number of rows) of the incidence matrix which is $(q^k-1)/(q-1)$. Solving the corresponding Diophantine system of equations is only possible for small dimensions. Therefore we apply a well-known method [1,20] to shrink this system by prescribing a group of automorphisms, i.e.

a subgroup of the general linear group $GL(k,q)$. This will be described in the next section. This method to reduce the size of the system of equations by prescribing automorphisms has since the first use in 1976 by Kramer and Mesner [21] been successfully applied in several cases like design theory [3,21], $q-$analoga of designs [9], arcs [7] and the construction of distance-optimal codes [5,6]. Already in [14] the author constructed new distance optimal codes by combining orbits.

#### Two-weight Codes with Prescribed Projective Groups

We no longer search for an arbitrary solution, which corresponds to a selection of columns of the generator matrix or equivalently to a selection of points in a projective point set. In the reduced system an entry equal to one in the first half of the solution corresponds to a selection of a complete orbit of points under the action (multiplication) of a subgroup $G$ of $GL(k,q)$. In the language of linear codes this means that the linear code has $G$ as a subgroup in its group of automorphisms. For the incidence matrix $M$ the selection of complete orbits of points corresponds to the addition of columns corresponding to the points in the orbit. This reduces the size of the matrix $M$ to one, where the number of columns is the number of orbits. The action of $G$ on the points induces an action on subspaces, and this action preserves incidence in the following sense:

**Lemma 3** *Denote by $B \cdot v$ the action of a matrix $B \in G < GL(k,q)$ on a subspace $v$ of $GF(q)^k$. Let $p$ be a point of $PG(k-1,q)$ and $H$ be a hyperplane in $PG(k-1,q)$. Then we have for all matrices $B$ :*

$$p \subset H \iff B \cdot p \subset B \cdot H.$$

Because of this property the rows labeled by the hyperplanes in an orbit of $G$ are identical after the column reduction. This shrinks the square matrix $M$ of size $(q^k-1)/(q-1)$ to a square matrix $M^G$ of a smaller size $m$. Where $m$ is the number of orbits. The rows of $M^G$ are labeled by the orbits $\Omega_1, \ldots, \Omega_m$ of $G$ on the hyperplanes, the columns are labeled by the orbits $\omega_1, \ldots, \omega_m$ of $G$ on the points. For an entry in $M^G$ we have

$$M^G_{i,j} = |\{p \in \omega_j : p \subset H_i\}|$$

for an arbitrary representative $H_i \in \Omega_i$. This allows us to give a version of the above corollary 2 in the case of a prescribed group of automorphisms.

**Theorem 4** *There is an $(n, k, n-w_1, n-w_2)$ point set in $PG(k-1,q)$ with a subgroup $G < GL(k,q)$ of automorphisms if and only if there is a $(0/1)-$solution $(x,y)$ of the Diophantine system of linear equations:*

$$(1)\ (M^G, J^G)(x,y)^T = \begin{pmatrix} n-w_1 \\ \vdots \\ n-w_1 \end{pmatrix}$$

$$(2) \quad \textstyle\sum_{i=1,\ldots,m} |\omega_i| \quad = \quad n$$

Such a solution is a vector of length $2m$ where $m$ is the number of orbits of $G$ on the points (resp. hyperplanes) in $PG(k-1,q)$. An entry $x_i$ equal to one says that the corresponding orbit is part of the $(n, k, n-w_1, n-w_2)$ point set. The $0/1$ distribution in $y$ says, like in the case of corollary 2, how many hyperplanes meet the points from the corresponding $i-$th orbit.

**Results**

To apply theorem 4 we need to know the two weights $w_1$ and $w_2$ of the linear code. As we are looking for projective codes, no two columns of the generator matrix are linearly dependent, so we know that the minimum distance of the dual code is at least $3$. This allows to use the first $3$ MacWilliams [1] identities to get candidates for the two weights. Using these candidates we apply theorem 4 for several subgroups of $GL(k,q)$. A last crucial step is the use of an effective algorithm [25] by A. Wassermann for the solution of the reduced Diophantine system of linear equations.

It is known [10,12] that a projective two-weight code can be used to define a strongly regular graph. A strongly regular graph is a $K-$regular graph with $N$ vertices and each pair of adjacent vertices has $\lambda$ common neighbors, and each pair of non-adjacent vertices has $\mu$ common neighbors.

In the following tables we give the parameters of the two-weight codes we found using our method together with parameters of the corresponding strongly regular graph. Our table extends the results in [15] which used backtracking algorithms to construct all possible generator matrices. They got all inequivalent projective two weight codes. We were able to compute some (not all) two-weight codes for larger parameters. The left part of the tables below gives the parameters of the code, the fourth column the minimum weight together with the number of codewords of this weight, the fifth column the same information for the second weight. The next four columns give the parameters $N, K, \lambda, \mu$ of the corresponding strongly regular graph. In the last column we give information on the found code, in the case of already known codes we give a citation or a method of construction already given in [10]. An entry 'known' or some reference different from a construction in [10] means that the corresponding strongly regular graph was known before. This does not necessarily mean that the code was already known.

Optimal codes (minimum distance meets some known upper bound) are marked with $*$.

The author thanks Andries Brouwer who helped to compare the parameters with his database of two-weight codes/strongly regular graphs.
We list the constructed two-weight codes for given $q, k$ up to $n = \left\lfloor \frac{q^k-1}{4} \right\rfloor$ which is half the number of all possible points. We do not list codes known from the SU2 construction of [10].

*Table 1: Binary codes of dimension* 8

| $n$ | $k$ | $q$ | $w_1$ | $w_2$ | $N$ | $K$ | $\lambda$ | $\mu$ | info |
|---|---|---|---|---|---|---|---|---|---|
| 51* | 8 | 2 | 24(204) | 32(51) | 256 | 51 | 2 | 12 | known |
| 60 | 8 | 2 | 24(60) | 32(195) | 256 | 60 | 20 | 12 | SU2,FE1 |
| 68* | 8 | 2 | 32(187) | 40(68) | 256 | 68 | 12 | 20 | FE1 |
| 85* | 8 | 2 | 40(170) | 48(85) | 256 | 85 | 24 | 30 | known |
| 102* | 8 | 2 | 48(153) | 56(102) | 256 | 102 | 38 | 42 | known |
| 119 | 8 | 2 | 56(136) | 64(119) | 256 | 119 | 54 | 56 | RT2 |
| 128* | 8 | 2 | 64(254) | 128(1) | 256 | 128 | 0 | 128 | CY2,SU1 |

*Table 2: Binary codes of dimension* 9

The codes for $n = 70$ and $n = 196$ were found by Bierbrauer/Edel in 1997. (update Aug. 07)

| $n$ | $k$ | $q$ | $w_1$ | $w_2$ | $N$ | $K$ | $\lambda$ | $\mu$ | info |
|---|---|---|---|---|---|---|---|---|---|
| 70* | 9 | 2 | 32(315) | 40(196) | 512 | 70 | 6 | 10 | [4] |
| 73* | 9 | 2 | 32(219) | 40(292) | 512 | 73 | 12 | 10 | [17] |
| 196* | 9 | 2 | 96(441) | 112(70) | 512 | 196 | 60 | 84 | [4] |
| 219 | 9 | 2 | 96(73) | 112(438) | 512 | 219 | 106 | 84 | [17] |
| 256* | 9 | 2 | 128(510) | 256(1) | 512 | 256 | 0 | 256 | SU1 |

*Table 3: Binary codes of dimension* 10

The code for $n = 198$ may be a new optimal two-weight code, the strongly regular graph was known.

| $n$ | $k$ | $q$ | $w_1$ | $w_2$ | $N$ | $K$ | $\lambda$ | $\mu$ | info |
|---|---|---|---|---|---|---|---|---|---|
| 198* | 10 | 2 | 96(825) | 112(198) | 1024 | 198 | 22 | 42 | new |
| 231* | 10 | 2 | 112(792) | 128(231) | 1024 | 231 | 38 | 56 | [13] |
| 264* | 10 | 2 | 128(759) | 144(264) | 1024 | 264 | 56 | 72 | FE1 |
| 297 | 10 | 2 | 144(726) | 160(297) | 1024 | 297 | 76 | 90 | [13] |
| 330 | 10 | 2 | 160(693) | 176(330) | 1024 | 330 | 98 | 110 | [13] |
| 363 | 10 | 2 | 176(660) | 192(363) | 1024 | 363 | 122 | 132 | [13] |
| 396 | 10 | 2 | 192(627) | 208(396) | 1024 | 396 | 148 | 156 | [13] |
| 429 | 10 | 2 | 208(594) | 224(429) | 1024 | 429 | 176 | 182 | [13] |
| 462 | 10 | 2 | 224(561) | 240(462) | 1024 | 462 | 206 | 210 | [13] |
| 495 | 10 | 2 | 240(528) | 256(495) | 1024 | 495 | 238 | 240 | known |
| 512 | 10 | 2 | 256(1022) | 512(1) | 1024 | 512 | 0 | 512 | SU1 |

*Table 4: Binary codes of dimension* 11

There are no new codes.

| $n$ | $k$ | $q$ | $w_1$ | $w_2$ | $N$ | $K$ | $\lambda$ | $\mu$ | info |
|---|---|---|---|---|---|---|---|---|---|
| 276 | 11 | 2 | 128(759) | 144(1288) | 2048 | 276 | 44 | 36 | RT5 |
| 759 | 11 | 2 | 352(276) | 384(1771) | 2048 | 759 | 310 | 264 | RT5d |
| 1024 | 11 | 2 | 512(2046) | 1024(1) | 2048 | 1024 | 0 | 1024 | SU1 |

*Table 5: Binary codes of dimension* 12

We list codes different from the construction SU2 . 'new' means that the strongly regular graph and the two weight code are both new.

| $n$ | $k$ | $q$ | $w_1$ | $w_2$ | $N$ | $K$ | $\lambda$ | $\mu$ | info |
|---|---|---|---|---|---|---|---|---|---|
| 234* | 12 | 2 | 112(2808) | 128(1287) | 4096 | 234 | 2 | 14 | FE3 |
| 270 | 12 | 2 | 128(2295) | 144(2184) | 4096 | 270 | 14 | 18 | known |
| 273 | 12 | 2 | 128(1911) | 144(2184) | 4096 | 273 | 20 | 18 | [22] |
| 455* | 12 | 2 | 224(3640) | 256(455) | 4096 | 455 | 6 | 56 | known |
| 780 | 12 | 2 | 384(3315) | 416(780) | 4096 | 780 | 116 | 156 | known |
| 845 | 12 | 2 | 416(3250) | 448(845) | 4096 | 845 | 144 | 182 | new |
| 910 | 12 | 2 | 448(3185) | 480(910) | 4096 | 910 | 174 | 210 | new |
| 975 | 12 | 2 | 480(3120) | 512(975) | 4096 | 975 | 206 | 240 | known |
| 1040 | 12 | 2 | 512(3055) | 544(1040) | 4096 | 1040 | 240 | 272 | known |
| 1105 | 12 | 2 | 544(2990) | 576(1105) | 4096 | 1105 | 276 | 306 | new |
| 1170 | 12 | 2 | 576(2925) | 608(1170) | 4096 | 1170 | 314 | 342 | [24] |
| 1300 | 12 | 2 | 640(2795) | 672(1300) | 4096 | 1300 | 396 | 420 | new |
| 1365 | 12 | 2 | 672(2730) | 704(1365) | 4096 | 1365 | 440 | 462 | known |
| 1430 | 12 | 2 | 704(2665) | 736(1430) | 4096 | 1430 | 486 | 506 | new |
| 1495 | 12 | 2 | 736(2600) | 768(1495) | 4096 | 1495 | 534 | 552 | new |
| 1560 | 12 | 2 | 768(2535) | 800(1560) | 4096 | 1560 | 584 | 600 | known |
| 1625 | 12 | 2 | 800(2470) | 832(1625) | 4096 | 1625 | 636 | 650 | new |
| 1690 | 12 | 2 | 832(2405) | 864(1690) | 4096 | 1690 | 690 | 702 | new |
| 1755 | 12 | 2 | 864(2340) | 896(1755) | 4096 | 1755 | 746 | 756 | new |
| 1800* | 12 | 2 | 896(3825) | 960(270) | 4096 | 1800 | 728 | 840 | known |
| 1820 | 12 | 2 | 896(2275) | 928(1820) | 4096 | 1820 | 804 | 812 | new |
| 1885 | 12 | 2 | 928(2210) | 960(1885) | 4096 | 1885 | 864 | 870 | new |
| 1911 | 12 | 2 | 896(273) | 960(3822) | 4096 | 1911 | 950 | 840 | [22] |
| 1950 | 12 | 2 | 960(2145) | 992(1950) | 4096 | 1950 | 926 | 930 | new |
| 2015 | 12 | 2 | 992(2080) | 1024(2015) | 4096 | 2015 | 990 | 992 | known |
| 2048 | 12 | 2 | 1024(4094) | 2048(1) | 4096 | 2048 | 0 | 2048 | SU1 |

*Table 6: Ternary codes of dimension* 5

| $n$ | $k$ | $q$ | $w_1$ | $w_2$ | $N$ | $K$ | $\lambda$ | $\mu$ | info |
|---|---|---|---|---|---|---|---|---|---|
| 11 | 5 | 3 | 6(132) | 9(110) | 243 | 22 | 1 | 2 | known |
| 55 | 5 | 3 | 36(220) | 45(22) | 243 | 110 | 37 | 60 | known |

*Table 7: Ternary codes of dimension* 6

There is no new two-weight code, but we mention references to the found codes.

| $n$ | $k$ | $q$ | $w_1$ | $w_2$ | $N$ | $K$ | $\lambda$ | $\mu$ | |
|---|---|---|---|---|---|---|---|---|---|
| 56 | 6 | 3 | 36(616) | 45(112) | 729 | 112 | 1 | 20 | FE2 |
| 84 | 6 | 3 | 54(560) | 63(168) | 729 | 168 | 27 | 42 | [18] |
| 98 | 6 | 3 | 63(532) | 72(196) | 729 | 196 | 43 | 56 | [18] |
| 112 | 6 | 3 | 72(504) | 81(224) | 729 | 224 | 61 | 72 | RT2 |
| 126 | 6 | 3 | 81(476) | 90(252) | 729 | 252 | 81 | 90 | RT2,FE1 |
| 140 | 6 | 3 | 90(286) | 99(280) | 729 | 280 | 103 | 110 | DeResmini |
| 154 | 6 | 3 | 99(420) | 108(308) | 729 | 308 | 127 | 132 | [19,14] |
| 168 | 6 | 3 | 108(392) | 117(336) | 729 | 336 | 153 | 156 | [23] |

We found no new ternary two-weight codes of dimension 7, the only parameters were those from the construction SU1.

*Table 8: Ternary codes of dimension* 8

'new' means that the strongly regular graph and the two weight code are both new.

| $n$ | $k$ | $q$ | $w_1$ | $w_2$ | $N$ | $K$ | $\lambda$ | $\mu$ | |
|---|---|---|---|---|---|---|---|---|---|
| $328^*$ | 8 | 3 | 216(5904) | 243(656) | 6561 | 656 | 7 | 72 | known |
| 656 | 8 | 3 | 432(5248) | 459(1312) | 6561 | 1312 | 223 | 272 | known |
| 738 | 8 | 3 | 486(5087) | 513(1476) | 6561 | 1476 | 297 | 342 | known |
| 820 | 8 | 3 | 540(4920) | 567(1640) | 6561 | 1640 | 379 | 420 | new |
| 902 | 8 | 3 | 594(4756) | 621(1804) | 6561 | 1804 | 469 | 506 | new |
| 984 | 8 | 3 | 648(4592) | 675(1968) | 6561 | 1968 | 567 | 600 | new |
| 1066 | 8 | 3 | 702(4428) | 729(2132) | 6561 | 2132 | 673 | 702 | known |
| 1107 | 8 | 3 | 729(4346) | 756(2214) | 6561 | 2214 | 729 | 756 | known |
| 1148 | 8 | 3 | 756(4264) | 783(2296) | 6561 | 2296 | 787 | 812 | [22] |
| 1189 | 8 | 3 | 783(4182) | 810(2378) | 6561 | 2378 | 847 | 870 | new |
| 1230 | 8 | 3 | 810(4100) | 837(2460) | 6561 | 2460 | 909 | 930 | new |
| 1271 | 8 | 3 | 837(4018) | 864(2542) | 6561 | 2542 | 973 | 992 | new |
| 1312 | 8 | 3 | 864(3936) | 891(2624) | 6561 | 2624 | 1039 | 1056 | known |
| 1353 | 8 | 3 | 891(3854) | 918(2706) | 6561 | 2706 | 1107 | 1122 | new |
| 1394 | 8 | 3 | 918(3772) | 945(2788) | 6561 | 2788 | 1177 | 1190 | new |
| 1435 | 8 | 3 | 945(3690) | 972(2870) | 6561 | 2870 | 1249 | 1260 | [22] |
| 1476 | 8 | 3 | 972(3608) | 999(2952) | 6561 | 2952 | 1323 | 1332 | known |
| 1517 | 8 | 3 | 999(3526) | 1026(3034) | 6561 | 3034 | 1399 | 1406 | new |
| 1558 | 8 | 3 | 1026(3444) | 1053(3116) | 6561 | 3116 | 1477 | 1482 | new |
| 1599 | 8 | 3 | 1053(3362) | 1080(3198) | 6561 | 3198 | 1557 | 1560 | new |

A more detailed version of these tables (together with generator matrices and the used group of automorphisms) can be found at the URL: http://linearcodes.uni-bayreuth.de/twoweight/.

**Concluding remark**

There is the construction SU2 of Calderbank/Kantor [10] which gives two-weight codes for a series of parameters. Looking at the results for the pairs $(q, k) = (2, 12), (3, 6), (3, 8)$ (see tables 5,7,8) suggests that

there might be a similar series sharing the same weights of the SU2-two-weight codes, but having different lengths of the codewords. For example in the case $q = 3$ and $k = 8$ the construction SU2 gives codes with weights $w_1, w_2$ and length $n$:

$$w_1 = 27 \cdot i, w_2 = 27 + w_1, n = 40 + 40 \cdot i \ \ (i = 1, \ldots, 40).$$

The above table 8 indicates that we found two-weight codes for nearly all pairs of weights of this SU2 series but for a different length $n = 41 \cdot i$ starting with $i = 8$.

**References**


[1] A. Betten, M. Braun, H. Fripertinger, A. Kerber, A. Kohnert and A. Wassermann: Error Correcting Linear Codes, Springer 2006.

[2] A. Betten, R. Laue and A. Wassermann: *Simple 7-Designs with Small Parameters*. Journal of Combinatorial Designs 7, pp. 79-94, 1999

[3] A. Betten, A. Kerber, A. Kohnert, R. Laue and A. Wassermann: *The discovery of simple 7-designs with automorphism group* $P\Gamma L(2, 32)$. Proceedings Applied Algebra, Algebraic Algorithms and Error-Correcting Codes. 11th international symposium, AAECC-11, Paris, France, July 17-22, 1995, Lect. Notes Comput. Sci. 948, pp. 131-145, 1995.

[4] J. Bierbrauer and Y. Edel: *A family of 2-weight codes related to BCH-codes*. J. Comb. Des. Vol. 5, pp. 391-396, 1997.

[5] M. Braun: *Construction of Linear Codes with Large Minimum Distance*. IEEE Transactions on Information Theory, Vol.50, pp. 1687-1691, 2004.

[6] M. Braun, A. Kohnert and A. Wassermann: *Optimal Linear Codes From Matrix Groups*, IEEE Transactions on Information Theory, Vol. 51, pp. 4247-4251, 2005.

[7] M. Braun, A. Kohnert and A. Wassermann: *Construction of (n,r)-arcs in PG(2,q)*, Innovations in Incidence Geometry 1, pp. 133-141, 2005.

[8] M. Braun: *Construction of a point-cyclic resolution in PG(9,2)*, Innovations in Incidence Geometry 3, pp. 33-50, 2006.

[9] M. Braun, A. Kerber and R Laue: *Systematic Construction of q-Analogs of Designs*, Des. Codes Cryptography 34, pp. 55-70, 2005

[10] R. Calderbank and W. M. Kantor, *The Geometry of Two-Weight Codes*, Bull. London Math. Soc. 18, pp. 97-122, 1986.

[11] F. De Clerk and M. Delanote, *Two-Weight Codes, Partial Geometries and Steiner Systems*, Des. Codes Cryptography 21, pp. 87-98, 2000.

[12] P. Delsarte, *Weights of linear codes and strongly regular normed spaces*, Discrete Math. 4, pp. 47-64, 1972.

[13] L. A. Dissett, Combinatorial and Computational Aspects of Finite Geometries, Ph. D. Thesis, University of Toronto, 2000.

[14] M. van Eupen, *Some New results for Ternary Linear Codes of Dimension* $5$ *and* $6$, IEEE Transactions on Information Theory, Vol.41, pp. 2048-2051, 1995.

[15] V. Fack, I. Bouyukliev, W. Willems and J. Winne, *Projective two-weight codes with small parameters and their corresponding graphs*, Proceedings OC2005, Pamporovo 2005, pp. 139-145, 2005.

[16] V. Fack, I. Bouyukliev, W. Willems and J. Winne, *Projective two-weight codes with small parameters and their corresponding graphs*, preprint 25p., August 2005.

[17] F. Fiedler and M.Klin, *A strongly regular graph with the parameters (512, 73, 438, 12, 10) and its dual graph*. Preprint. MATH-AL-7-1998, Technische Universität Dresden, July 1998, 23 pp.

[18] T. A. Gulliver, *Two new optimal ternary two-weight codes and strongly regular graphs*, Discrete Math. 149, pp. 83-92, 1996.

[19] T. A. Gulliver, *A new two-weight code and strongly regular graph*, Appl. Math. Lett. 9, pp. 17-20, 1996.

[20] A. Kerber: *Applied Finite Group Actions*, Springer, 1999.

[21] E. S. Kramer and D. M. Mesner, *t-Designs on Hypergraphs*, Discrete Math. 15, pp. 263-296, 1976.

[22] C. L. M. de Lange, *Some New Cyclotomic Strongly Regular Graphs*, J. Algebr. Comb. 4, pp. 329 - 330, 1995.

[23] T. Penttila and G. F. Royle, *Sets of type* $(m, n)$ *in the affine and projective planes of order nine*. Des. Codes Cryptography 6, pp. 229-245, 1995.

[24] M. de Resmini and G. Migliori, *A* $78-$*set of Type* $(2, 6)$ *in* $PG(2, 16)$, Ars Comb. 22, pp. 73 - 75, 1986.

[25] A. Wassermann: *Finding Simple* $t$*-Designs with Enumeration Techniques*, Journal of Combinatorial Designs 6, pp. 79-90, 1998.