Abstract

Perfect coloings of the infinite rectangular grid

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Let $M = (m_{ij})_{i,j=1}^{n}$ be an arbitrary nonnegative integer matrix. A coloring of vertices of a graph G into n colors is called *perfect* with the matrix M if the number of vertices of a color j incident to a vertex of a color i does not depend on the vertex and equals to m_{ij}. We consider perfect colorings of the graph $G(Z^2)$, that is an infinite rectangular grid. This graph is regular of degree 4. Each vertex of the graph $G(Z^2)$ corresponds to a pair of integers, two vertices are adjacent if their pairs differ in one coordinate by unit, and the other coordinate is the same. We say that a matrix M is *admissible*, if a perfect coloring of the infinite rectangular grid with matrix M exists.

A coloring of a graph G can be considered as a function

$$\varphi : V(G) \to \{1, ..., n\}.$$

A perfect coloring φ of $G(Z^2)$ is (p, q)-*periodic* if $\varphi(x + p, y + q) = \varphi(x, y)$ for any integers x, y. A perfect coloring is called *periodic* if it is (p, p)- and $(q, -q)$-periodic. We prove that a periodic perfect coloring exists for any admissible matrix.

All perfect colorings into three colors are classified and all corresponding admissible matrices are presented. Similar problem was solved for two colors by Axenovich in 2003.