Abstract

Line spreads of polar spaces of rank 4 inducing generalized quadrangles

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Let Π be a polar space of rank 4 over a field $\mathbb{K}$ such that the generalized quadrangle $Res^+(\alpha)$ of a line α of Π which consists of the planes and 3-spaces of Π containing α, admits a spread. Let $\mathcal{L}$ be a line-spread of Π with the following property:

Let $\mathcal{D}$ be the set of 3-spaces of Π in which $\mathcal{L}$ induces spreads. For every point Σ of Π, the 3-spaces of $\mathcal{D}$ containing Σ all contain the spread line $\lambda \in \mathcal{L}$ covering Σ and form a spread of the generalized quadrangle $Res^+(\lambda)$.

Given such a spread $\mathcal{L}$, we show that $\Gamma = (\mathcal{L}, \mathcal{D})$ is a generalized quadrangle which we characterize for the classical polar spaces $\Pi \cong Sp_8(\mathbb{K})$ and $O^-_{10}(\mathbb{K})$ as $Sp_4(\mathbb{K}(\zeta))$ and $H_5(\mathbb{K}(\zeta))$, respectively, where $\mathbb{K}(\zeta)$ is a quadratic field extension of $\mathbb{K}$. For finite polar spaces, we show they are the only two admitting such a spread. We give an example of a spread $\mathcal{L}$ for the infinite hermitian polar space $H_8(\mathbb{C})$ over the complex numbers $\mathbb{C}$ where $\Gamma = (\mathcal{L}, \mathcal{D})$ is a hermitian generalized quadrangle $H_4(\mathbb{Q})$ over the quaternions.

This research is motivated by the following: Dualizing Π, the point set $\bigcup_{X \in \mathcal{D}} X^{\perp_\Delta}$ of the dual polar space Δ dual of Π is a hyperplane of Δ intersecting each symp Σ, i.e. an element of maximal type of Δ, in the neighbours of an ovoid of a quad of Σ.