Abstract

Reconstruction of permutations from their erroneous patterns

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Reconstruction problem arises in graph theory and in coding theory as well as in molecular biology if one is interested in reconstructing unknown genetic sequences. Efficient reconstruction of sequences was introduced and investigated by Levenshtein in the 90's for combinatorial channels with errors of essential interests in coding theory such as substitutions, transpositions, deletions and insertions of symbols. We continue these investigations to solve the reconstruction problem of permutations and signed permutations on n elements from their erroneous patterns which are distorted by reversal of intervals (with replacing signs in the case of signed permutations).

We show that for any $n \geq 2$ an unknown signed permutation is uniquely reconstructible from 3 signed permutations being at the reversal distance at most one from an unknown signed permutation. The reversal distance is defined as the minimal number of reversals of an permutation interval which are needed to transform one permutation into another. Under the same conditions for any $n \geq 3$ an unknown permutation is uniquely reconstructible from 4 permutations. We also investigate the cases when a smaller number of permutations or signed permutations are sufficient to determine an unknown permutation or singed permutation uniquely.

The proposed approach is based on an investigation of structural properties of a certain graph constructed for this problem. In particular, it is proved that the considered graph for signed permutations does not contain C_3, C_5 and bipartite subgraphs $K_{2,3}$ and contains C_4. The considered graph for permutations does not contain C_3 and bipartite subgraphs $K_{2,4}$ and contains bipartite subgraphs $K_{3,3}$.

It is also shown that in the case of at most two reversal errors it is needed much more different erroneous patterns to reconstruct an unknown permutation or signed permutation.