Abstract

The Hermitian variety $\mathrm{H}(5,4)$ has no ovoid

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(joint work with Klaus Metsch)

We consider the Hermitian varieties $\mathrm{H}(2n+1,q^2)$. An ovoid $\mathcal{O}$ is a set of points of $\mathrm{H}(2n+1,q^2)$ such that every generator of $\mathrm{H}(2n+1,q^2)$ meets the set $\mathcal{O}$ in exactly one point.

A lot of research has been done to prove the existence or non-existence of ovoids of classical polar spaces. One of the open cases is the Hermitian variety $\mathrm{H}(2n+1,q^2)$. For $n=1$, every Hermitian curve $\mathrm{H}(2,q^2)$ contained in $\mathrm{H}(3,q^2)$ constitutes an ovoid of $\mathrm{H}(3,q^2)$, and even different examples can be found. On the other hand, no Hermitian variety $\mathrm{H}(2n+1,q^2)$, $n\geq 2$, having ovoids is known. Furthermore, it is known ([3]) that $\mathrm{H}(2n+1,q^2)$, $q=p^h$, p prime, has no ovoid when

$$p^{2n+1} > \binom{2n+p}{2n+1}^2 - \binom{2n+p-1}{2n+1}^2.$$

From this it follows that for each prime p there exists an integer n_p such that $\mathrm{H}(2n+1,q^2)$, with $n\geq n_p$, has no ovoid. All obtained integers n_p are larger than two. Thus, for no polar space $\mathrm{H}(5,q^2)$, the problem on the existence of an ovoid has been solved. We present a combinatorial approach, using similar arguments as in [2], that shows that $\mathrm{H}(5,4)$ has no ovoid.

References

[1] J. De Beule and K. Metsch. The Hermitian variety $\mathrm{H}(5,4)$ has no ovoid. *Bull. Belgian Math. Soc. Simon Stevin*. In *Proceedings of Incidence Geometry Conference (La Roche, Belgium, 2004)*, to appear.

[2] A. Klein. Partial Ovoids in Classical Finite Polar Spaces. *Des. Codes Cryptogr.*, 31:221–226, 2004.

[3] G. Eric Moorhouse. Some p-ranks related to Hermitian varieties. *J. Statist. Plann. Inference*, 56(2):229–241, 1996. Special issue on orthogonal arrays and affine designs, Part II.